Least Common Multiple, Lowest Common Denominator, and Comparing Fractions

Least Common Multiple
The least common multiple (LCM) of two numbers is the smallest number that is a multiple of both numbers. There are several methods for finding the least common multiple of two or more numbers.

Method #1
One method of finding the LCM is by listing multiples of each number.

Example 1
Find the least common multiple of 4 and 10 by listing multiples.

4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, …

10, 20, 30, 40, 50, 60, 70, 80, 90, 100, …

There are many common multiples of 4 and 10 such as 20, 40 and 60.
The common multiples are infinite.
The first common multiple 20 is the least common multiple.

Answer: 20

Example 2
Find the least common multiple of 15, 40, and 60.

15, 30, 45, 60, 75, 90, 105, 120, 135, …

40, 80, 120, 160, …

60, 120, 180, …

The least common multiple is 120.

Answer: 120

Method #2
You can also find the LCM by writing the prime factorization of each number.

Example 3
Find the least common multiple of 15, 40 and 60.

15 = 3 × 5   *3 occurs once as a factor.*
                                                    5 occurs once as a factor.

40 = 2 × 2 × 2 × 5   *2 occurs three times as a factor.*
                                                    5 occurs once as a factor.

60 = 2 × 2 × 3 × 5   *2 occurs twice as a factor.*
                                               3 occurs once as a factor.
                                               5 occurs once as a factor.

Write each factor the greatest number of times that it occurs in any of the numbers.

Now multiply these factors to find the LCM.

The greatest number of times that 2 occurs as a factor is three times.
The greatest number of times that 3 occurs as a factor is once.
The greatest number of times that 5 occurs as a factor is once.

2 × 2 × 2 × 3 × 5 = 120

The least common multiple is 120.

Answer: 120

Example 4

Find the least common multiple of 49, 63 and 20 by writing the prime factorization of each number.

49 = 7 × 7   *7 occurs twice as a factor.*

63 = 3 × 3 × 7   *3 occurs twice as a factor.*
                                               7 occurs once as a factor.

20 = 2 × 2 × 5   *2 occurs twice as a factor.*
                                               5 occurs once as a factor.

Write each factor the greatest number of times that it occurs in any of the numbers.

Now multiply these factors to find the LCM.

The greatest number of times that 2 occurs as a factor is twice.
The greatest number of times that 3 occurs as a factor is twice.
The greatest number of times that 5 occurs as a factor is once.
The greatest number of times that 7 occurs as a factor is twice.

2 × 2 × 3 × 3 × 7 × 7 × 5 = 8820

The greatest common factor is 8820.

Answer: 8,820

Method #3

A third method of finding the LCM of numbers involves dividing by primes.

Remember the primes are: 2, 3, 5, 7, 11, 13, 17, …

1. Write the numbers in a row and divide by the first prime number that will divide exactly into at least two of the numbers. Write the prime number divisor on the left of the numbers.

2. Bring down the quotients and any number not divisible by the prime to the next row below.

3. Divide by the same prime number again if possible and then find the next prime divisor. Continue the process until no two numbers are divisible by any prime number.

4. The least common multiple is the product of all the prime divisors and the numbers in the final row.

Example 5

Find the least common multiple of 4 and 10 by dividing by primes.

> Find the first prime number that will divide exactly into 4 and 10.

2	4	10
	2	5

Divide 4 and 10 by the first prime number 2.

Write the result below the numbers.

2	4	10
	2	5

> *No prime number will divide into the numbers 2 and 5 on the bottom row.*

> *Find the product of the prime number divisor and the numbers on the bottom row. This product will be the least common multiple.*

> *2 × 2 × 5 = 20*

> *The LCM is 20.*

> **Answer: 20**

Example 6

Find the least common multiple of 15, 40 and 60 by dividing by primes.

> Find the first prime number that will divide exactly into at least two of the number.

If a number is not divisible by the prime number, bring it down to the next row.
Continue until no prime number will divide into two numbers.
Remember to divide only by primes.

2	15	40	60
	15	20	30

Divide 40 and 60 by the first prime number 2.

Write the result below the numbers.

Bring down 15 since it is not divisible by 2.

2	15	40	60
2	15	20	30
	15	10	15

Divide again by the prime number 2.

Divide 20 and 30 by 2.

Bring down 15 since it is not divisible by 2.

2	15	40	60
2	15	20	30
3	15	10	15
	5	10	5

Next divide by 3.

Divide 15 and 15 by 3.

Bring down 10 since it is not divisible by 3.

2	15	40	60
2	15	20	30
3	15	10	15
5	5	10	5
	1	2	1

Next divide by 5.

Divide 5 and 10 and 5 by 5.

Now, no prime number will divide into two numbers on the bottom row, so the process is done.

Find the product of the prime number divisors and the numbers on the bottom row.

2 × 2 × 3 × 5 × 1 × 2 × 1 = 120

The LCM is 120.

Answer: 120

Choosing a Method

You can find the LCM by any of the three methods.

If the first method of listing multiples requires a very long list of multiples, switch to one of the methods using prime numbers.

Writing fractions with the Lowest Common Denominator

The lowest common denominator (LCD) is the least common multiple (LCM).

Example 7

Find the lowest common denominator of the following fractions and rewrite the fractions using the lowest common denominator.

$\frac{2}{3}$ and $\frac{4}{5}$

List multiples of 3 and 5.

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, …

5, 10, 15, 20, 25, 30, …

The lowest common denominator is 15.

Change each denominator to 15.

$$\frac{2}{3} = \frac{?}{15}$$

$$\frac{4}{5} = \frac{?}{15}$$

$$\frac{2 \times 5}{3 \times 5} = \frac{10}{15} \quad \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$$

Answer: $\frac{10}{15}$ and $\frac{12}{15}$

Comparing Fractions

To compare fractions you must have a common denominator.

Example 8

Compare the following fractions:

$\frac{2}{3}$ and $\frac{4}{7}$

List multiples of 3 and 7.

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, …

7, 14, 21, 28, 35, 42, 49, 56, …

The lowest common denominator is 21.

Change each denominator to 21.

$$\frac{2}{3} = \frac{?}{21} \text{ and } \frac{4}{7} = \frac{?}{21}$$

$$\frac{2 \times 7}{3 \times 7} = \frac{14}{21} \text{ and } \frac{4 \times 3}{7 \times 3} = \frac{12}{21}$$

$$\frac{14}{21} > \frac{12}{21} \text{ so } \frac{2}{3} > \frac{4}{7}$$

Answer: $\frac{2}{3} > \frac{4}{7}$