Order of Operations with Fractions

Order of Operations

1. Do all operations inside parentheses and other grouping symbols.

2. Evaluate exponents.

3. Multiply and divide from left to right.

4. Add and subtract from left to right.

Example 1

Solve. $\frac{1}{3} \cdot \frac{2}{3}^2 - \frac{1}{9}$

$\frac{1}{3} \cdot (\frac{2}{3} \cdot \frac{2}{3}) - \frac{1}{9}$

$\frac{1}{3} \cdot \frac{4}{9} - \frac{1}{9}$

$\frac{4}{27} - \frac{1}{9}$

$\frac{4}{27} - \frac{3}{27}$

Answer: $\frac{1}{27}$

There are no operations inside parentheses so evaluate the exponents first.

Multiply next.
Always multiply before adding or subtracting.
Now subtract. Remember to get a common denominator when subtracting.

Example 2

Solve. $\frac{1}{4}^3 + \frac{3}{4} \cdot \frac{1}{2}$

$\frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} + \frac{3}{4} \cdot \frac{1}{2}$

$\frac{1}{64} + \frac{3}{4} \cdot \frac{1}{2}$

$\frac{1}{64} + \frac{3}{8}$

$\frac{1}{64} + \frac{24}{64}$

Answer: $\frac{25}{64}$

There are no operations inside parentheses so evaluate the exponents first.

Multiply next.
Always multiply before adding or subtracting.
Find a common denominator
Add the fractions.

Example 3

Solve. $\frac{3}{5} + \frac{2}{3} - \;\; \frac{1}{5} + \frac{1}{3}$ *Perform operations inside parentheses first.*

$\frac{3}{5} + \frac{2}{3} - \;\; \frac{1}{5} + \frac{1}{3}$ *Find a common denominator when*
adding fractions.

$\frac{3}{5} + \frac{2}{3} - \;\; \frac{3}{15} + \frac{5}{15}$

$\frac{3}{5} + \frac{2}{3} - \;\; \frac{8}{15}$ *Now add and subtract from left to right.*

$\frac{9}{15} + \frac{10}{15} - \frac{8}{15}$ *Find a common denominator for all*
the denominators.

$\frac{19}{15} - \frac{8}{15}$

Answer: $\frac{11}{15}$

Example 4

Solve. $9 \div \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{2} - 1$ *Multiply and divide from left to right.*

$\frac{9}{1} \cdot \frac{3}{1} \cdot \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{2} - 1$ *When dividing, invert and multiply.*

$\frac{27}{1} \cdot \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{2} - 1$

$\frac{27}{3} \cdot \frac{1}{3} + \frac{1}{2} - 1$

$\frac{27}{9} + \frac{1}{2} - 1$ *Add and subtract from left to right.*
Get a common denominator.

$\frac{54}{18} + \frac{9}{18} - \frac{18}{18}$

$\frac{63}{18} - \frac{18}{18}$

$\frac{45}{18}$

$2\frac{9}{18}$

Answer: $2\frac{1}{2}$

Example 5

Solve. $\frac{3}{5} - \frac{1}{2} \cdot \frac{1}{3}$ *Always multiply before adding or subtracting.*

$\frac{3}{5} - \frac{1}{6}$ *Find a common denominator when*
subtracting fractions.

$\frac{18}{30} - \frac{5}{30}$

Answer: $\frac{13}{30}$

Example 6

Solve. $\frac{1}{3} \div \frac{7}{5} - \frac{1}{7}$ *Always divide before adding or subtracting.*

$\frac{1}{3} \cdot \frac{5}{7} - \frac{1}{7}$ *When dividing, invert and multiply.*

$\frac{5}{21} - \frac{1}{7}$ *Get a common denominator when*
subtracting fractions.

$\frac{5}{21} - \frac{3}{21}$

Answer: $\frac{2}{21}$

Example 7

Solve. $\frac{1}{2} - \frac{1}{4} + \frac{3}{8} \cdot \frac{1}{5} - \frac{1}{8}$

$\frac{1}{2} - \frac{1}{4} + \mathbf{\color{red}\frac{3}{8} \cdot \frac{1}{5}} - \frac{1}{8}$ *Multiply before adding or subtracting.*

$\mathbf{\color{red}\frac{1}{2} - \frac{1}{4}} + \frac{3}{40} - \frac{1}{8}$ *Add or subtract from left to right.*

$\mathbf{\color{red}\frac{2}{4} - \frac{1}{4}} + \frac{3}{40} - \frac{1}{8}$ *Get a common denominator when subtracting.*

$\mathbf{\color{red}\frac{1}{4} + \frac{3}{40}} - \frac{1}{8}$ *Get a common denominator when adding.*

$\frac{10}{40} + \frac{3}{40} - \frac{1}{8}$

$\frac{13}{40} - \frac{1}{8}$

$\frac{13}{40} - \frac{5}{40}$

$\frac{8}{40}$ *Reduce.*

Answer: $\frac{1}{5}$

Example 8

Solve. $\dfrac{7}{8} \cdot \dfrac{3}{5} - \left(\dfrac{1}{5} + \dfrac{1}{4}\right) + \left(\dfrac{3}{2}\right)^2$

$$\dfrac{7}{8} \cdot \dfrac{3}{5} - \left(\dfrac{1}{5} + \dfrac{1}{4}\right) + \left(\dfrac{3}{2}\right)^2$$

Perform operations inside the parentheses first.

$$\dfrac{7}{8} \cdot \dfrac{3}{5} - \left(\dfrac{4}{20} + \dfrac{5}{20}\right) + \left(\dfrac{3}{2}\right)^2$$

Get a common denominator when adding fractions.

$$\dfrac{7}{8} \cdot \dfrac{3}{5} - \dfrac{9}{20} + \left(\dfrac{3}{2}\right)^2$$

Next evaluate the exponents.

$$\dfrac{7}{8} \cdot \dfrac{3}{5} - \dfrac{9}{20} + \dfrac{3}{2} \cdot \dfrac{3}{2}$$

$$\dfrac{7}{8} \cdot \dfrac{3}{5} - \dfrac{9}{20} + \dfrac{9}{4}$$

Next multiply before adding or subtracting.

$$\dfrac{21}{40} - \dfrac{9}{20} + \dfrac{9}{4}$$

$$\dfrac{21}{40} - \dfrac{18}{40} + \dfrac{90}{40}$$

Get a common denominator when adding or subtracting fractions.

$$\dfrac{21}{40} - \dfrac{18}{40} + \dfrac{90}{40}$$

Next add or subtract from left to right.

$$\dfrac{3}{40} + \dfrac{90}{40}$$

$$\dfrac{93}{40}$$

$$2\dfrac{13}{40}$$

Example 9

The following rainfall amounts were recorded: June $2\frac{1}{2}$ inches, July $3\frac{1}{4}$ inches, August $4\frac{1}{5}$ inches and September $4\frac{3}{4}$ inches. Find the average rainfall for the four months.

To find the average amount of rainfall in this problem, add the numbers and divide by the number of numbers. So you will add the four numbers and divide by 4.

$$2\frac{1}{2} = \frac{10}{20}$$

$$3\frac{1}{4} = \frac{5}{20}$$

$$4\frac{1}{5} = \frac{4}{20}$$

$$+\ 4\frac{3}{4} = \frac{15}{20}$$

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$$13\frac{34}{20}$$

$$13\ +\ 1\frac{14}{20}$$

$$14\frac{14}{20}$$

$$14\frac{7}{10}$$ *Now divide this sum by 4.*

$$14\frac{7}{10} \div 4$$

$$\frac{147}{10} \div \frac{4}{1}$$

$$\frac{147}{10} \cdot \frac{1}{4}$$

$$\frac{147}{40} = 3\frac{27}{40}$$

The average rainfall is $3\frac{27}{40}$ inches.

Answer: $3\frac{27}{40}$

Example 10

Bob weighed $145\frac{1}{2}$ pounds, Bill weighed 180 and John weighed $162\frac{3}{4}$ pounds.

Find the average weight for the three boys.

To find the average weight in this problem, add the numbers and divide by the number of numbers. So you will add the three numbers and divide by 3.

$$145\ \tfrac{1}{2} = \ \tfrac{4}{8}$$

$$180$$

$$+\ 162\ \tfrac{3}{4}\ =\ \tfrac{6}{8}$$

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$$487\ \tfrac{10}{8}$$

$$487\ +\ 1\tfrac{2}{8}$$

$$487\ +\ 1\tfrac{1}{4}$$

$$488\tfrac{1}{4} \qquad\qquad\qquad\qquad\qquad\text{\textit{Now divide this sum by 3.}}$$

$$488\tfrac{1}{4} \div 3$$

$$\frac{1953}{4} \div \frac{3}{1}$$

$$\frac{1953}{4} \cdot \frac{1}{3}$$

$$\frac{1953}{12} = 162\tfrac{3}{4}$$

The average weight is $162\frac{3}{4}$ pounds

Answer: $162\frac{3}{4}$