The Addition and Subtraction Properties of Equality

Linear equation

A linear equation in one variable can be written as follows.

$ax + b = c$ *a, b and c are any real numbers*.

Example 1

The following equations are examples of linear equations.

- $2x + 8 = 16$
- $5x = -50$
- $x - 6 = 10$

A solution of an equation is a replacement for the variable that makes the equation true.

Example 2

Find the solution to the following equation.

$x + 3 = 10$

The solution of this equation is 7 since replacing x with 7 will make the equation true.

$$x + 3 = 10$$
$$7 + 3 = 10$$
$$10 = 10$$

The properties of equality help us find a solution to an equation.

Addition Property of Equality

Let a, b, and c represent any real numbers.

You can add the same number to both sides of an equation and get an equivalent equation.

$a = b$

$a + c = b + c$

Subtraction Property of Equality

Let a, b, and c represent any real numbers.

You can subtract the same number from both sides of an equation and get an equivalent equation.

$a = b$

$a - c = b - c$

When solving an equation for x, we must get x alone. We can do this by performing the inverse operation. Addition and subtraction are inverse operations. We can add the same number to each side of an equation or subtract the same number from each side of an equation.

Example 3

$x + 6 = 10$ *The inverse of adding 6 is subtracting 6.*

$x + 6 - 6 = 10 - 6$ *Subtract 6 from each side of the equation.*

$x + 0 = 4$

Answer: $\boldsymbol{x = 4}$

Example 4

$x - 8 = 12$ *The inverse of subtracting 8 is adding 8. Add 8 to each side of the equation.*

$x - 8 + 8 = 12 + 8$ *Adding opposites will give a sum of zero.*

$-8 + 8 = 0$

$x + 0 = 20$

Answer: $\boldsymbol{x = 20}$

Example 5

$x + 4.5 = 10.8$ *The inverse of adding 4.5 is subtracting 4.5.*

$x + 4.5 - 4.5 = 10.8 - 4.5$ *Subtract 4.5 from each side of the equation.*

$x + 0 = 6.3$

Answer: $\boldsymbol{x = 6.3}$

Example 6

$x - \frac{1}{3} = \frac{3}{4}$

The inverse of subtracting $\frac{1}{3}$ is adding $\frac{1}{3}$. Add $\frac{1}{3}$ to each side of the equation.

$x - \frac{1}{3} + \frac{1}{3} = \frac{3}{4} + \frac{1}{3}$

Adding opposites will give a sum of zero.
$-\frac{1}{3} + \frac{1}{3} = 0.$

When adding fractions, get a common denominator. $\frac{3}{4} + \frac{1}{3}$

$\frac{3\ x\ 3}{4\ x\ 3} = \frac{9}{12}$ and $\frac{1\ x\ 4}{3\ x\ 4} = \frac{4}{12}$

$x + 0 = \frac{9}{12} + \frac{4}{12}$

$x = \frac{13}{12}$

Answer: $\boldsymbol{x = 1\frac{1}{12}}$

Example 7

$20 = x - 9$

Sometimes x is on the right side of the equation. The inverse of subtracting 9 is adding 9.

$20 + 9 = x - 9 + 9$

Add 9 to each side of the equation.

$29 = x + 0$

Answer: $\boldsymbol{29 = x}$

This is the same as $x = 29$.

Example 8

$5 = x + 8$

Here x is on the right side of the equation. The inverse of adding 8 is subtracting 8.

$5 - 8 = x + 8 - 8$

Subtract 8 from each side of the equation.

$-3 = x + 0$

Answer: $\boldsymbol{-3 = x}$

This is the same as $x = -3$.

Combine like terms

Sometimes we need to simplify one or both sides of an equation before using the addition or subtraction property of equality. Combine like terms on each side of the equation.

Example 9

$4x + 6 - 3x - 3 = 15$ — *Combine like terms on the left side of the equation.*

$4x - 3x + 6 - 3 = 15$ — *Change the order. Combine the x variables.*

$1x + 6 - 3 = 15$ — *Combine the numbers.*

$1x + 3 = 15$

$1x + 3 - 3 = 15 - 3$ — *Subtract 3 from each side of the equation.*

$1x + 0 = 12$

$1x = 12$

Anwer: $x = 12$

Example 10

$2.3 = -7c + 0.5 + 8c + 0.1$ — *Combine like terms on the right side of the equation.*

$2.3 = -7c + 8c + 0.5 + 0.1$ — *Combine the c variables.*

$2.3 = 1c + 0.5 + 0.1$ — *Combine the decimal numbers.*

$2.3 = 1c + 0.6$ — *Now get c alone on the right side of the equation.*

$2.3 - 0.6 = 1c + 0.6 - 0.6$ — *Subtract 0.6 on each side of the equation.*

$1.7 = 1c + 0$

Answer: $1.7 = c$

The Opposite of x

A minus sign in front of x can be read as the opposite of x.

Let a represent any real number.

- $-x = a$ — *The opposite of x is a.*
- $x = -a$ — *Therefore x equals the opposite of a.*

Example 11

$-x = 8$ — *The opposite of x is 8.*

Answer: $x = -8$ — *Therefore $x = -8$.*

Example 12

$-x = -5$ — *The opposite of x is -5.*

Answer: $x = 5$ — *Therefore $x = 5$.*

Variables on Both Sides of the Equation

Sometimes variables are on both sides of the equation. We need to get the variables on one side of the equation and numbers on the other side. You can get the variables alone on either side of the equation.

Example13

$8x + 5 = 7x$ — *Here the x variable appears on both sides of the equation.*

You must eliminate the x variable on one side of the equation.

One way to do this is to subtract $7x$ on each side.

$8x - 7x + 5 = 7x - 7x$ — *Notice that $7x - 7x = 0$.*

$1x + 5 = 0$ — *Now x is only on the left side of the equation. Next, get x alone.*

$1x + 5 - 5 = 0 - 5$ — *Subtract 5 on each side of the equation.*

$1x + 0 = -5$

$1x = -5$

Answer: $x = -5$

There is often more than one way to solve a problem. If you use the addition and subtraction properties correctly, you will always get the correct answer.

Example 14

We could work the problem in example 13 another way.
We could eliminate the variable on the left side of the equation.

$8x + 5 = 7x$

$8x - 8x + 5 = 7x - 8x$ — *Subtract $8x$ on each side of the equation.*

$0 + 5 = -1x$ — *Now x is just on the right side of the equation.*

$5 = -x$ — *This says that 5 equals the opposite of x.*

Answer: $-5 = x$ — *Therefore -5 equals x.*

This is the same solution we found in example 13. You can eliminate the variable from either side of the equation and still get the same answer.

Eliminate the Parentheses in an Equation

Example15

$5(x - 1) = 6x + 4$ — *Distribute to get rid of the parentheses.*

$5x - 5 = 6x + 4$

$5x - 6x - 5 = 6x - 6x + 4$ — *Eliminate the x variable on the right side of the equation.*

$-1x - 5 = 0 + 4$
$-1x - 5 = 4$ — *Now get x alone on the left side of the equation. Add 5 to each side of the equation. Don't try to eliminate the coefficient of x which is -1 before adding 5 to each side of the equation.*

$-1x - 5 + 5 = 4 + 5$ — *Add 5 to each side of the equation.*

$-1x + 0 = 9$

$-1x = 9$ — *Remember: $-1x = -x$*

$-x = 9$ — *This means the opposite of x equals 9.*

Answer: $x = -9$ — *Therefore x equals -9.*

Example 16

$-6x + 8 = 10 - 5x + 14$

$-6x + 8 = 10 + 14 - 5x$ — *Combine like terms on the right side of the equation.*

$-6x + 8 = 24 - 5x$ — *Eliminate the x variable on the right side of the equation.*

$-6x + 5x + 8 = 24 - 5x + 5x$ — *Add $5x$ to each side of the equation.*

$-1x + 8 = 24 + 0$

$-1x + 8 = 24$ — *Now get x alone.*

$-1x + 8 - 8 = 24 - 8$ — *Subtract 8 from each side of the equation.*

$-1x + 0 = 16$

$-1x = 16$ — *Remember: $-1x = -x$*

$-x = 16$ — *This means the opposite of x equals 16.*

Answer: $x = -16$ — *Therefore x equals -16.*

Example 17

$9x - 3 - 5x = -3x + 8 + 6x$

$9x - 5x - 3 = -3x + 8 + 6x$ — *Combine like terms on the left side of the equation.*

$4x - 3 = -3x + 6x + 8$ — *Combine like terms on the right side of the equation.*

$4x - 3 = 3x + 8$ — *Eliminate $3x$ on the right side of the equation.*

$4x - 3x - 3 = 3x - 3x + 8$ — *Subtract $3x$ on each side of the equation.*

$1x - 3 = 0 + 8$

$1x - 3 = 8$

$1x - 3 + 3 = 8 + 3$ — *Add 3 to each side of the equation.*

$1x + 0 = 11$

$1x = 11$

Answer: $x = 11$

Example 18

$8(a+2)-10-a=6(a+1)$ — *Distribute to eliminate the parentheses.*

Multiply by the number outside the parentheses.

$8a+16-10-a=6a+6$ — *Combine terms on the left side of the equation.*

$8a-a+16-10=6a+6$ — *Combine the a variables.*

$8a-1a+16-10=6a+6$ — *Remember:* $-a=-1a$

$7a+16-10=6a+6$

$7a+6=6a+6$ — *Eliminate the variable on the right side of the equation.*

$7a-6a+6=6a-6a+6$ — *Subtract* $6a$ *on each side of the equation.*

$1a+6=0+6$

$1a+6=6$ — *Now get a alone on the left side of the equation.*

$1a+6-6=6-6$ — *Subtract 6 on each side of the equation.*

$1a+0=0$

$1a=0$

Answer: $\boldsymbol{a=0}$

Some equations have no solution and some have an infinite number of solutions. If the variable term is eliminated on both sides of the equation and you end up with a false statement, there is no solution to the equation. If you end up with a true statement, there are infinite solutions to the equation and we say the equation has all real numbers for the solution.

Example 19

$3x + 7 + 1 = 2x + 1x + 4 + 4$

$3x + 7 + 1 = 2x + 1x + 4 + 4$	*Combine like terms.*
$3x + 8 = 2x + 1x + 8$	*Combine like terms.*
$3x + 8 = 3x + 8$	
$3x - 3x + 8 = 3x - 3x + 8$	*Subtract $3x$ on each side of the equation.*
$8 = 8$	*The variables are eliminated on each side of the equation.*

$8 = 8$ is a true statement so the solution is all real numbers.

Example 20

$3(x + 4) = 3x + 10 + 1$

$3(x + 4) = 3x + 10 + 1$	*Distribute on the left side of the equation.*
	Combine like terms on the right side of the equation.
$3x + 12 = 3x + 11$	
$3x - 3x + 12 = 3x - 3x + 11$	*Subtract $3x$ from each side of the equation.*
$12 = 11$	*The variables are eliminated from each side of the equation.*

$12 = 11$ is a false statement. There is no solution.

Example 21

$5x = 5x$

$5x - 5x = 5x - 5x$	*Subtract $5x$ from each side of the equation.*
$0 = 0$	*The variables are eliminated from each side of the equation.*

$0 = 0$ is a true statement so the solution is all real numbers.

Example 22

$5 + 2(x + 2) = 2x + 9$

$5 + 2(x + 2) = 2x + 9$	*Remember the order of operations.*
	Multiply (distribute) before adding.
$5 + 2x + 4 = 2x + 9$	*Collect like terms.*
$2x + 9 = 2x + 9$	
$2x - 2x + 9 = 2x - 2x + 9$	*Subtract $2x$ from each side of the equation.*
$9 = 9$	*The variables are eliminated from each side of the equation.*

$9 = 9$ is a true statement so the solution is all real numbers.
