Problem Solving: Motion, Mixture, and Investment Problems

Motion Problems

Example 1

The distance traveled is 576 miles. The time is 8 hours. Find the speed (rate).

Distance = Rate x Time

576 = R x 8
72 = R
The rate is 72 miles per hour.

Example 2

The rate traveled is 50 miles per hour and the distance traveled is 450 miles. Find the time.

Distance = Rate x Time

450 = 50 x T
9 = T
The time traveled is 9 hours.

Finding the time when one object catches up with another object

Example 3

A bus leaves a station traveling 50 mph. A car traveling 60 mph leaves the same station three hours later. How long will it take the car to overtake the bus?

A table is often useful when solving a motion problem. Identify the given information and the unknown.

The rate of the bus is 50 mph and the rate of the car is 60 mph.

We are finding the amount of time that it will take the car to overtake the bus. We will call this amount of time x.

Since the bus left the station 3 hours earlier than the car, the bus has been traveling 3 hours longer. Therefore the time the bus has been on the road is x + 3. Next fill the appropriate numbers in the chart.

	Distance =	**Rate x**	**Time**
Bus		50	x + 3
Car		60	x

Now we use the distance formula to solve the problem. Distance = rate x time.
The bus distance = 50(x+3) and the car distance = 60x.

	Distance =	**Rate x**	**Time**
Bus	50(x + 3)	50	x + 3
Car	60x	60	x

At the point in time when the car catches up with the bus, the car and bus will have traveled the same distance down the road. So set their distances equal to solve the problem.

Bus $\longrightarrow$

Car $\longrightarrow$

50(x + 3) = 60x
50x + 150 = 60x
50x – 50x + 150 = 60x – 50x
150 = 10x
Answer: 15= x
The car will have to travel 15 hours to catch up with the bus.

Finding the time when one object catches up with another object

Example 4

John leaves on a train traveling 45 miles per hour. His wife leaves 1 hour later in a car traveling 70 miles per hour. How long will it take the car to overtake the train?Identify the given information and the unknown.

The rate of the train is 45 mph and the rate of the car is 70 miles per hour.

We are finding the amount of time it will take the car to overtake the train. We will call this x.

Since the train left first, an hour earlier, it has been on the road an hour longer than the car. Therefore the number of hours the train has been on the road is x + 1. Next fill in the appropriate numbers in the chart.

	Distance =	**Rate x**	**Time**
Train		45	x + 1
Car		70	x

Now we use the distance formula to solve the problem. Distance = rate x time.
The train distance = 45(x + 1) and the car distance = 70x.

	Distance =	**Rate x**	**Time**
Train	45(x + 1)	45	x+1
Car	70x	70	x

At the point in time when the car catches up with the train, the car and train will have traveled the same distance down the road. So set their distances equal to solve the problem.

Train $\longrightarrow$

Car $\longrightarrow$

45(x+1) = 70x
45x + 45 = 70x
45x – 45x + 45 = 70x - 45x
45 = 25x
Answer: 1.8 = x
It takes the car 1.8 hours to catch up with the train.

Finding the time when two objects traveling in opposite directions cross paths

Example 5

A bus leaves town A traveling toward town B. At the same time a car leaves town B traveling toward town A. The speed of the bus is 50 miles per hour and the speed of the car is 60 miles per hour. The distance between the two towns is 2000 miles. How many hours until the bus and car meet?
Identify the given information and the unknown.

The rate of the bus is 50 mph and the rate of the car is 60 miles per hour.

Since the bus and the car left at the same time they have been traveling the same amount of time. We call this time x. Next fill the appropriate numbers in the chart.

	Distance =	Rate x	Time
Bus		50	x
Car		60	x

Now we use the distance formula to solve the problem. Distance = rate x time.
The train distance = 50x and the car distance = 60x.

	Distance =	Rate x	Time
Bus	50x	50	x
Car	60x	60	x

The total distance between town A and town B is 2,000 miles. At the time when the bus and car cross paths, they have traveled a total of 2,000 miles which is the distance between the towns. The sum of their distances is 2,000 miles.

Distance the Bus has traveled + Distance the Car has traveled = 2000

$$50x + 60x = 2000$$
$$110x = 2000$$
$$x = 18.2$$

The time is 18.2 hours rounded to the nearest tenth.
The bus and car will cross paths after 18.2 hours.

Finding the time when two objects traveling in opposite directions cross paths

Example 6

Bob leaves his house traveling toward John's house driving 65 miles per hour. An hour later, John leaves his house traveling on the same road toward Bob's house. John is traveling 50 miles per hour. It is 500 miles between their houses. How many hours before Bob and John meet? Identify the given information and the unknown.

Bob's rate is 65 mph and John's rate is 50 mph.

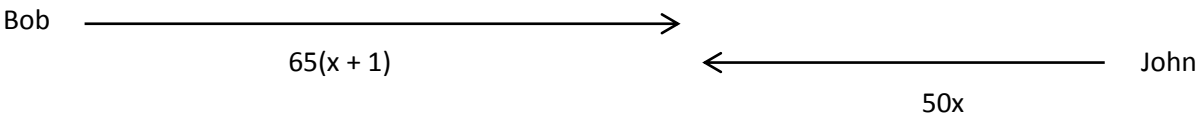
Since the Bob left his house first he has been traveling on the road longer than John. Bob has been traveling one hour longer than John. Bob's time is x + 1 and John's time is x. Next fill the appropriate numbers in the chart.

	Distance =	**Rate x**	**Time**
Bob		65	x + 1
John		50	x

We use the distance formula to solve the problem. Distance = rate x time.
Bob's distance is 65(x + 1) and John's distance is 50x.

	Distance =	**Rate x**	**Time**
Bob	65(x + 1)	65	x + 1
John	50x	50	x

The total distance between Bob's house and John's house is 500 miles. At the time when Bob and John cross paths, they have traveled a total of 500 miles. The sum of their distances is 500.



Distance Bob has traveled + Distance John has traveled = 500

$$65(x + 1) + 50x = 500$$
$$65x + 65 + 50x = 500$$
$$115x + 65 = 500$$
$$115x = 435$$
$$x = 3.8$$

The time is 3.8 hours rounded to the nearest tenth.
Bob and John will cross paths after 3.8 hours.

Distance Problem

Example 7

John travels from home to the city. His speed going to the city is 60 mph. When he returns home by the same route his speed is 40 mph. The total trip takes 5 hours. What is the distance from his home to the city? Identify the given information and the unknown.

John's rate going to the city is 60 mph and his rate returning home is 40 mph.

Let x represent the time it takes to travel to the city. Since the total traveling time is 5 hours and x hours are used to travel to the city, then 5 – x hours are left to travel home. Notice the expression is 5 – x and not x – 5. Next fill the appropriate numbers in the chart.

	Distance =	**Rate x**	**Time**
Home to city		60	x
City to home		40	5 - x

Use the distance formula to solve the problem. Distance = rate x time.

The distance from home to the city is 60x.
The distance from the city to home is 40(5 – x).

	Distance =	**Rate x**	**Time**
Home to city	60x	60	x
City to home	40(5 - x)	40	5 - x

Home to City $\longrightarrow$

City to Home $\longleftarrow$

Since John is going from home to the city and then back on the same route his distance each way is the same.

Distance Home to City = Distance City to Home

$$60x = 40(5 - x)$$
$$60x = 200 - 40x$$
$$100x = 200$$
$$x = 2$$

The time it would take him to go from home to the city would be 2 hours.
The question asks for the distance from his home to the city.

We must find the distance from John's home to the city.
The distance from his home to city is 60x and we found that x = 2.

The distance = 60(2) = 120 miles.
The distance from John's home to the city is 120 miles.

Mixture Problem

Example 8

How many ounces of 30% acid should be mixed with 40 ounces of 2% acid to obtain a 12% acid solution?

Use a chart to organize the given and unknown information.
We use the following formula: % x ounces = Amount of Acid.

We have 40 ounces of 2% acid.
We have x ounces of 30% acid.
In the final solution we want 12% acid.

Since we are mixing 40 ounces with x ounces, the final solution will have x + 40 ounces.

Fill the information into the chart.

	% x	**Ounces =**	**Amount of Acid**
30% solution	0.30	x	
2% solution	0.02	40	
Final solution 12%	0.12	(x + 40)	

Next multiply (percent times ounces) to get the amount of acid.

0.30 times unknown ounces = 0.30x
0.02 times 40 ounces = 0.02(40)
0.12 times (x + 40) ounces = 0.12(x + 40) = 0.12x + 4.8

	% x	**Ounces =**	**Amount of Acid**
30% solution	0.30	x	0.30x
2% solution	0.02	40	0.02(40)
Final solution 12%	0.12	(x + 40)	0.12x + 4.8

30% solution + 2% solution = 12% final solution

$$0.30x + 0.02(40) = 0.12(x + 40)$$
$$0.30x + 0.8 = 0.12x + 4.8$$
$$0.30x - 0.12x + 0.8 = 0.12x - 0.12x + 4.8$$
$$0.18x + 0.8 = 4.8$$
$$0.18x + 0.8 - 0.8 = 4.8 - 0.8$$
$$0.18x = 4.0$$
$$x = 22.2$$

The answer is 22.2 rounded to the nearest tenth.
You must mix 22.2 ounces of 30% with 40 ounces of 2% to get a 12% solution.

Mixture Problem

Example 9

How many pounds of spice tea selling at $8 per pound should be mixed with an herbal tea selling at $2 per pound to produce 40 pounds selling at $3 per pound?

Spice tea and herbal tea are mixed together to make a total of 40 pounds.

If you let x represent the number of pounds of spice tea, then 40 – x is left for the number of pounds of herbal tea. Notice the expression is 40 – x and not x – 40.

Fill the given information into the chart.

	lbs x	**Price per lb =**	**Cost**
Spice tea	x	$8	
Herbal tea	40 - x	$2	
Final Mixture	40	$3	

Next multiply (pounds times price per pound) to get cost.

x lbs times $8 per pound = 8x
(40 – x) lbs times $2 per pound = 2(40 – x)
40 lbs times $3 per pound = 40(3)

	lbs x	**Price per lb =**	**Cost**
Spice tea	x	$8	8x
Herbal tea	40 - x	$2	2(40 – x)
Final Mixture	40	$3	40(3)

Cost of spice tea + cost of herbal tea = Cost of final mixture of tea

$$8x + 2(40 - x) = 40(3)$$
$$8x + 80 - 2x = 120$$
$$6x + 80 = 120$$
$$6x = 40$$
$$x = 6.7$$

x = 6.7 rounded to the nearest tenth.

You would mix 6.7 pounds of spice tea with (40 –x) or (40 – 6.7) = 33.3 pounds of herbal tea to produce the final mixture.

Investment Problem

Example 10

Bill has invested in two savings accounts. One earns 5% and the other earns 8%. The total amount invested is $2,100 and the total interest earned for the year is $153. How much is invested in each account?

The amount of money invested is called the principal.
The percent of interest is called the rate.

The total amount of money invested is $2,100.
Let x represent the amount of money invested at 5%.

Since the total amount of money invested is $2,100 and x dollars are used for the 5% account, (2100 – x) would be the amount of money left to be invested at 8%.

Notice it would be (2100 – x) and not (x – 2100).

Fill the information in the table.

	Principal x	**Rate =**	**Interest**
Saving Account 5%	x	0.05	
Savings Account 8%	2100 - x	0.08	
Total			

Multiply the principal times the rate to find the amount of interest earned in one year.

Formula: Principal x Rate = Interest

Interest at 5% 0.05x
Interest at 8% 0.08(2,100 – x)
The total amount of interest earned in a year was given to be $153.

	Principal x	**Rate =**	**Interest**
Saving Account 5%	x	0.05	0.05x
Savings Account 8%	2100 - x	0.08	0.08(2100 - x)
Total			153

Savings at 5% Interest + Saving at 8% Interest = Total Interest earned in one year

$$0.05x + 0.08(2100 - x) = 153$$
$$0.05 + 168 - 0.08x = 153$$
$$-0.03x + 168 = 153$$
$$-0.03x = -15$$
$$x = 500$$

$500 is invested in the savings at 5%.
(2100 – x) = 2100 - 500 = 1,600 invested in savings at 8%.

Investment Problem

Example 11

Julie has invested in two savings accounts. One earns 10% and the other earns 15%. She invests $200 more in the account that earns 15%. The total interest earned for one year is $230. How much is invested in each account?

Let x represent the amount of money invested at 10%

Since there is $200 more invested at 15%, let (x + 200) represent the amount invested at 15%.

Fill the information in the table.

	Principal x	**Rate =**	**Interest**
Saving Account 10%	x	0.10	
Savings Account 15%	x + 200	0.15	
Total			

Multiply the principal times the rate to find the amount of interest earned in one year.

Formula: Principal x Rate = Interest

Interest at 10% 0.10x
Interest at 15% 0.15(x + 200)
The total amount of interest earned for a year is $230.

	Principal x	**Rate =**	**Interest**
Saving Account 10%	x	0.10	0.10x
Savings Account 15%	x + 200	0.15	0.15(x + 200)
Total			230

Savings at 10% Interest + Saving at 15% Interest = Total Interest earned in one year

$$0.10x + 0.15(x + 200) = 230$$
$$0.10x + 0.15x + 30 = 230$$
$$0.25x + 30 = 230$$
$$0.25x = 200$$

$$x = 800$$

$800 is invested in the savings at 10%.

Since (x + 200) is invested at 15%, (800 + 200) or 1,000 is invested in savings at 15%.
