8.2.1 Confidence Intervals Finding $z_{\frac{\alpha}{2}}$

Students will be able to:

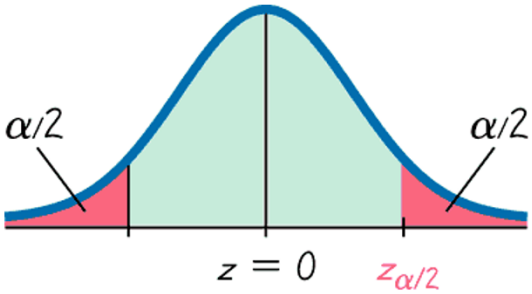
- Find the z-score given the confidence level

To estimate a population proportion, p, or population mean, μ, with a known population standard deviation, σ, we use a standard normal distribution to calculate the margin of error and construct the confidence interval.

The margin of error (and ultimately the width of the confidence interval) depends on the z-score that corresponds to the desired confidence level.

The confidence level is the area in the middle of the standard normal distribution.

The z-score has an area to the right of $\frac{\alpha}{2}$ is denoted by $z_{\frac{\alpha}{2}}$. This is the z-score we need for calculating a confidence interval.



The most common confidence levels and their corresponding z-scores are shown in the table.

CL	80%	90%	95%	98%	99%
α	0.20	0.10	0.05	0.02	0.01
$z_{\frac{\alpha}{2}}$	$z_{0.10}$	$z_{0.05}$	$z_{0.025}$	$z_{0.01}$	$z_{0.005}$
z	1.282	1.645	1.960	2.326	2.576

Note: A smaller α leads to a greater confidence level and a wider confidence interval.

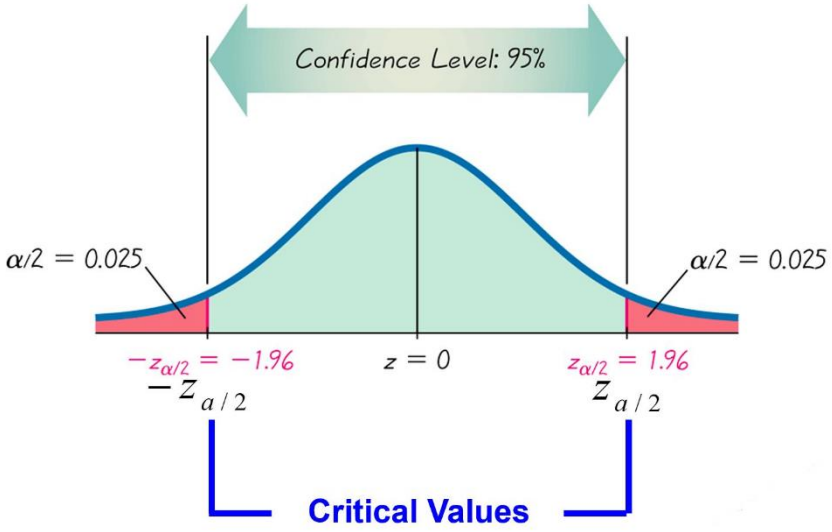
For example, let's say we want to find the z-score that corresponds with a 95% confidence level. First, find α.

$$\alpha = 1 - 0.95$$
$$= 0.05$$

So, $\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$

We look for the z-score that corresponds to an area of 0.025 to the right.

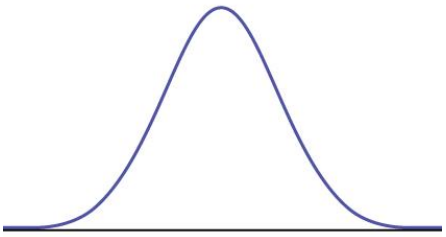
$$z_{0.025} = 1.960$$



To complete a confidence interval, we need to be able to determine the margin of error which has the following formula.

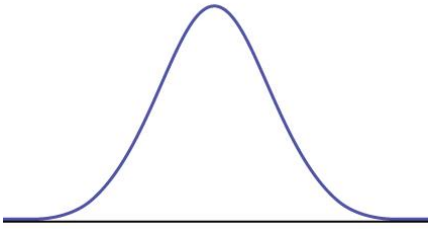
$$E = z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$$

Example: Suppose the number of hours employees work in a week at a company are normally distributed with an unknown population mean and a population standard deviation of 2 hours. A random sample of 37 employees' weekly hours produces a sample mean time of $\bar{x} = 39$ hours. What value of z should be used to calculate the confidence interval with a 90% confidence level?



Example: Find $z_{\frac{\alpha}{2}}$ that corresponds to

a) an 86% confidence level.



b) a 78% confidence level.